REPORT 1055

COMPARISON OF THEORETICAL AND EXPERIMENTAL HEAT-TRANSFER CHARACTERISTICS OF BODIES OF REVOLUTION AT SUPERSONIC SPEEDS [1]

By Richard Scherrer

SUMMARY

An investigation of the three important factors that determine convective heat-transfer characteristics at supersonic speeds, location of boundary-layer transition, recovery factor, and heat-transfer parameter has been performed at Mach numbers from 1.49 to 2.18. The bodies of revolution that were tested had, in most cases, laminar boundary layers, and the test results have been compared with available theory. Boundary-layer transition was found to be affected by heat transfer. Adding heat to a laminar boundary layer caused transition to move forward on the test body, while removing heat caused transition to move rearward. These experimental results and the implications of boundary-layer-stability theory are in qualitative agreement. Theoretical and experimental values of the recovery factor, based on the local Mach number just outside the boundary layer, were found to be in good agreement for both laminar and turbulent boundary layers on both of the body shapes that were investigated. In general, values of the heat-transfer parameter (Nusselt number divided by the square root of the Reynolds number) as determined for both heated and cooled cones with uniform and nonuniform surface temperatures, were in good agreement with theory. It was also found that the theory for cones could be used to predict the values of heat-transfer parameter for a pointed body of revolution with large negative pressure gradients with good accuracy.

INTRODUCTION

Aerodynamic heating of a body flying at high speeds results from the viscous dissipation of kinetic energy in the boundary layer. Although the viscosity of air is quite low, the increase in surface temperature due to aerodynamic heating becomes large at supersonic speeds. At speeds as low as 1,000 feet per second under conditions where the ambient temperature is above 40° F, the temperature rise is sufficient to make cabin cooling a necessity for man-carrying aircraft. At higher speeds the temperature rise can become large enough to cause serious structural problems. Although some natural cooling by radiation can be expected, continuous flight of airplanes and missiles at supersonic speeds requires some form of surface or internal temperature control.

In order to design cooling systems or adequate insulation for high-speed aircraft, the heat-transfer coefficients and the reference surface temperature (recovery temperature) must be known. The basic variables, the recovery temperature, and the Nusselt number are primarily functions of Mach number and Reynolds number. In addition to being influenced by the Mach number and Reynolds number, the Nusselt number is affected by a variety of other factors, the most important of which are surface-temperature gradients, longitudinal surface-pressure gradients, and body shape.

The recovery temperature and heat-transfer characteristics of bodies with laminar boundary layers are markedly different from those with turbulent boundary layers and for this reason the extent of the laminar-flow region in the boundary layer must be known. Because the over-all rates of heat transfer are, in general, less for laminar than for turbulent boundary layers, a body with a laminar boundary layer would be heated less rapidly than one with a turbulent boundary layer. This fact is of particular importance in the design of very high-speed missiles, and information on conditions which control the extent of the laminar-flow region in the boundary layer is needed. The theoretical work by Lees, reported in reference 1, indicates that heat transfer has a marked effect on the stability of laminar boundary layers on smooth flat plates; as a result, it is expected that there is a corresponding marked effect of heat transfer on the location of transition. Lees' theory indicates that the effect of transferring heat from a surface to a laminar boundary layer is to decrease the damping of small disturbances in the boundary layer, whereas extracting heat tends to increase the damping of small disturbances. The theory also indicates that at each supersonic Mach number a rate of heat extraction from a laminar boundary layer greater than a certain value results in complete damping of any small disturbances. This is of importance in the design of supersonic aircraft because of the possibility of extending the laminar region of the boundary layer.

The laminar boundary layer in compressible flow has proven to be much more amenable to theoretical treatment than the turbulent boundary layer. For this reason, a large

[1] Supersedes NACA RM A8L28, "Heat-Transfer and Boundary-Layer Transition on a Heated 20° Cone at a Mach Number of 1.53" by Richard Scherrer, William R. Wimbrow, and Forrest E. Gowen, 1949; NACA TN 1975, "Experimental Investigation of Temperature Recovery Factors on Bodies of Revolution at Supersonis Speeds" by William R. Wimbrow, 1949; NACA TN 2087, "Comparison of Theoretical and Experimental Heat Transfer on a Cooled 20° Cone with a Laminar Boundary Layer at a Mach Number of 2.02" by Richard Scherrer and Forrest E. Gowen, 1950; NACA TN 2131, "Boundary-Layer Transition on a Cooled 20° Cone at Mach Numbers of 1.5 and 2.0" by Richard Scherrer, 1950; and NACA TN 2148, "Laminar-Boundary-Layer Heat-Transfer Characteristics of a Body of Revolution with a Pressure Gradient at Supersonic Speeds" by William R. Wimbrow and Richard Scherrer, 1950.

body of theory exists for the laminar case while only a few investigators have developed heat-transfer theories for the turbulent case. For an example of the latter, see reference 2.

The heat-transfer literature devoted to temperature-recovery factors, reviewed in reference 3, indicates that theoretically the recovery factor for laminar boundary layers is equal to the square root of the Prandtl number, and, for turbulent boundary layers, is approximately equal to the cube root of the Prandtl number. Because in theory the recovery factors at moderate Mach numbers (1.0 to 3.0) are largely determined by the type of boundary layer, little variation would be expected between the recovery factors measured on flat plates, cones, and bodies with longitudinal pressure gradients.

The theory for determining heat transfer in the laminar boundary layer on a smooth, flat plate with a uniform surface temperature has been developed by both Crocco, and Hantshe and Wendt (reference 4).[2] The work of Crocco has been extended by Chapman and Rubesin (reference 5) to include the effects of nonuniform surface temperatures. The effects of body shape and longitudinal surface pressure gradients on the heat-transfer characteristecs of laminar boundary layers have been investigated theoretically in references 6 and 7. Because of the additional variables considered, the results of these two investigations are complex and therefore are difficult to apply. In reference 8, Mangler has presented relations whereby the characteristics of laminar boundary layers on any body of revolution can be determined if the characteristics of the laminar boundary layer on a related two-dimensional body are known. These relations can be used to determine the heat-transfer characteristics of a cone from those of a flat plate, but beyond this application the relations are somewhat restricted because of the difficulty in determining laminar boundary-layer characteristics on two-dimensional bodies with pressure gradients. For a body of revolution with negative pressure gradients (ogive, parabolic arc, etc.) one can infer that the boundary layer, and hence the heat transfer, is affected by two factors not affecting the boundary layer on cones. One is the decreasing rate of change of circumference with length, and the other is the pressure gradient. Since the effects of these two factors on the heat transfer tend to be compensating, it is possible that the theory for laminar boundary layers on cones may be used to predict the heat transfer in the negative-pressure-gradient regions of other bodies of revolution.

The present investigation was undertaken to determine the applicability of the available theoretical results of boundary-layer transition, recovery factor, and heat-transfer parameter. The purposes of the experimental phase of the investigation are:

1. To determine if heat transfer, both to and from a surface, affects boundary-layer transition.
2. To measure recovery factor on cones and on a body with a negative longitudinal pressure gradient for both laminar and turbulent boundary layers.
3. To measure the heat transfer in laminar boundary layers on cones with uniform and nonuniform surface temperatures.
4. To measure the heat transfer in a laminar boundary layer on a body with a negative longitudinal pressure gradient

[2] The work of Crocco, Hantsche and Wendt, and others has been conventently summarized by Johnson and Rubesin in reference 4.

Tests were conducted at several Mach numbers from 1.49 to 2.18 on three bodies of revolution under the following conditions:

1. Uniform surface pressure and a uniform temperature, heated surface (20° cone).
2. Uniform surface pressure and a uniform temperature, cooled surface (20° cone).
3. Uniform surface pressure and a nonuniform temperature, heated surface (20° cone, modified).
4. Nonuniform surface pressure and a uniform temperature, heated surface (parabolic-arc body).

Although the theory for laminar boundary layers indicates no basic change in heat-transfer processes between heated and cooled surfaces, it was considered desirable to obtain data under both conditions of heat flow in order to have the comparison of theory and experiment cover a broad range of surface temperature.

NOTATION

The following symbols have been used in the presentation of the theoretical and experimental data:

A	surface area of an increment of body length, square feet
C_d	skin-friction drag coefficient based on surface area, dimensionless
c_p	specific heat at constant pressure, Btu per pound, °F
C_r	temperature recovery factor, $\left(\frac{T_r-T_s}{T_0-T_s}\right)$, dimensionless
g	gravitational acceleration, 32.2 feet per second squared
H	wind-tunnel total pressure, pounds per square inch absolute
h	local heat-transfer coefficient $\left[\frac{Q}{A(T_r-T_s)}\right]$, Btu per second, square foot, °F
k	thermal conductivity, Btu per second, square foot, °F per foot
l	total body length, feet
M	Mach number, dimensionless
Nu	local Nusselt number $\left(\frac{h_s}{k_v}\right)$, dimensionless
p	static pressure, pounds per square inch absolute
Pr	Prandtl number $\left(\frac{c_p\mu_v}{k_v}\right)$, dimensionless
Q	total rate of heat transfer for an increment of body length, Btu per second
q	local rate of heat transfer, Btu per second, square foot
Re	local Reynolds number $\left(\frac{\rho_v Vs}{\mu_v}\right)$, dimensionless
Re_l	free-stream Reynolds number based on body length $\left(\frac{\rho_a Ul}{\mu_a}\right)$
r	body radius, feet
s	distance from nose along body surface, feet
T	temperature, °F
ΔT	temperature increment, °F
T_r	recovery temperature (surface temperature at condition of zero heat transfer), °F

$T_r^*=\dfrac{T_r+460}{T_v+460}$

$T_s^*=\dfrac{T_s+460}{T_v+460}$

T_{s_∞} theoretical surface temperature for infinite boundary-layer stability, °F
U free-stream velocity, feet per second
V velocity just outside the boundary layer, feet per second
W flow rate of coolant, pounds per second
x distance from the nose along the body axis, feet
x^* length ratio $\left(\dfrac{x}{l}\right)$, dimensionless
μ viscosity, pounds per foot, second
ρ air density, pounds per cubic foot

SUBSCRIPTS

In addition, the following subscripts have been used:

a fluid conditions in free stream
c coolant
0 stagnation conditions in the free stream
s conditions at the body surface
v fluid conditions just outside the boundary layer
w wind-tunnel wall

APPARATUS AND TESTS

Heat-transfer tests were conducted with three bodies: a heated 20° cone, a cooled 20° cone, and a heated parabolic-arc body. The two conical bodies were also used to investigate the effect of heat transfer on boundary-layer transition. The surface of each of the three models was ground and polished until the maximum roughness was less than 20 microinches in an attempt to eliminate surface finish as a variable in the investigation. All of the heat-transfer tests were similar in that measurements of the test conditions, local surface temperature T_s, and the incremental rates of heat transfer Q were made with each body. The recovery temperature, T_r, was measured in each set of experiments with the test-body heating or cooling system shut off and after both the wind tunnel and surface temperatures had reached equilibrium. The dimensionless heat-transfer parameter ($Nu/Re^{1/2}$, for laminar boundary layers) was determined from the experimental data by use of the following equation:

$$\frac{Nu}{Re^{1/2}}=\frac{Qs}{Ak_v(T_s-T_r)}\left(\frac{\mu_v}{\rho_v Vs}\right)^{1/2} \tag{1}$$

WIND TUNNEL

The tests with all of the models were performed in the Ames 1- by 3-foot supersonic wind tunnel No. 1. This closed-circuit, continuous-operation wind tunnel is equipped with a flexible-plate nozzle that can be adjusted to give test-section Mach numbers from 1.2 to 2.4. Reynolds number variation is accomplished by changing the absolute pressure in the tunnel from one-fifth of an atmosphere to approximately three atmospheres—the upper limit on pressure being a function of Mach number and ambient temperature. The water content of the air in the wind tunnel is maintained at less than 0.0001 pound of water per pound of dry air in order to eliminate humidity effects in the nozzle. All of the bodies tested were mounted in the center of the test section on sting-type supports. The total temperature of the air stream was measured by nine thermocouples in the wind-tunnel settling chamber.

HEATED CONE TESTS

Test body.—The details of the heated cone are shown in figure 1, and a photograph of the cone in the wind tunnel is shown in figure 2. The shell was machined from stainless steel because of its high electrical resistance, and all other parts were made of copper. Several longitudinal wall-thickness distributions, obtained by remachining the inside surface of the cone, were used to obtain different longitudinal surface-temperature distributions. The cone was heated by passing a high-amperage alternating electrical current (800 amperes maximum) at low voltage (0.45 volts maximum) longitudinally through the cone shell. Because the electrical resistance of the solid nose of the cone was very small, the forward 25 percent of the cone was, in effect, unheated

Instrumentation.—The wiring diagram of the cone, showing the electrical heating circuit and the temperature- and power-measurement apparatus, is shown in figure 3. Nine iron-constantan thermocouples were installed at equal length increments along the cone to determine the surface-temperature distributions. The thermocouples were installed in holes drilled completely through the shell and were soldered in place. The ten voltage leads were installed in a similar manner and were arranged on the selector switch so as to measure the voltage drop of each successive increment along the cone. These measurements, with the current measurements, determined the local heat input to the cone.

Test procedure.—Data were obtained at length Reynolds numbers of approximately 2.5 and 5.0 million at Mach numbers of 1.50 and 1.99. Cone temperatures of 30° F and 60° F above the wind-tunnel total temperature were selected as nominal values at which to obtain data. The surface temperatures were set to the desired values by adjusting the input voltage, and, with the cone at the desired temperature, the following data were recorded: air stream total temperature and pressure, test-section static pressure, air stream humidity, current, incremental voltages, and surface temperatures.

Accuracy.—The accuracy of the experimental data was determined by estimating the uncertainty of the individual measurements which entered into the determination of the final results. The estimated uncertainties of the basic measurements are as follows:

Total temperature, T_0	±1.5° F
Surface temperature, T_s	±0.5° F
Total pressure, H	±0.01 psia
Static pressure, p	±0.01 psia
Current	±2 percent
Incremental voltages	±2 percent
Cone dimensions	±0.005 inch

The maximum probable error of any given parameter was obtained from the component uncertainties by the method described in reference 9. The maximum probable error of the heat-transfer parameter $Nu/Re^{1/2}$ is ±6.8 percent and

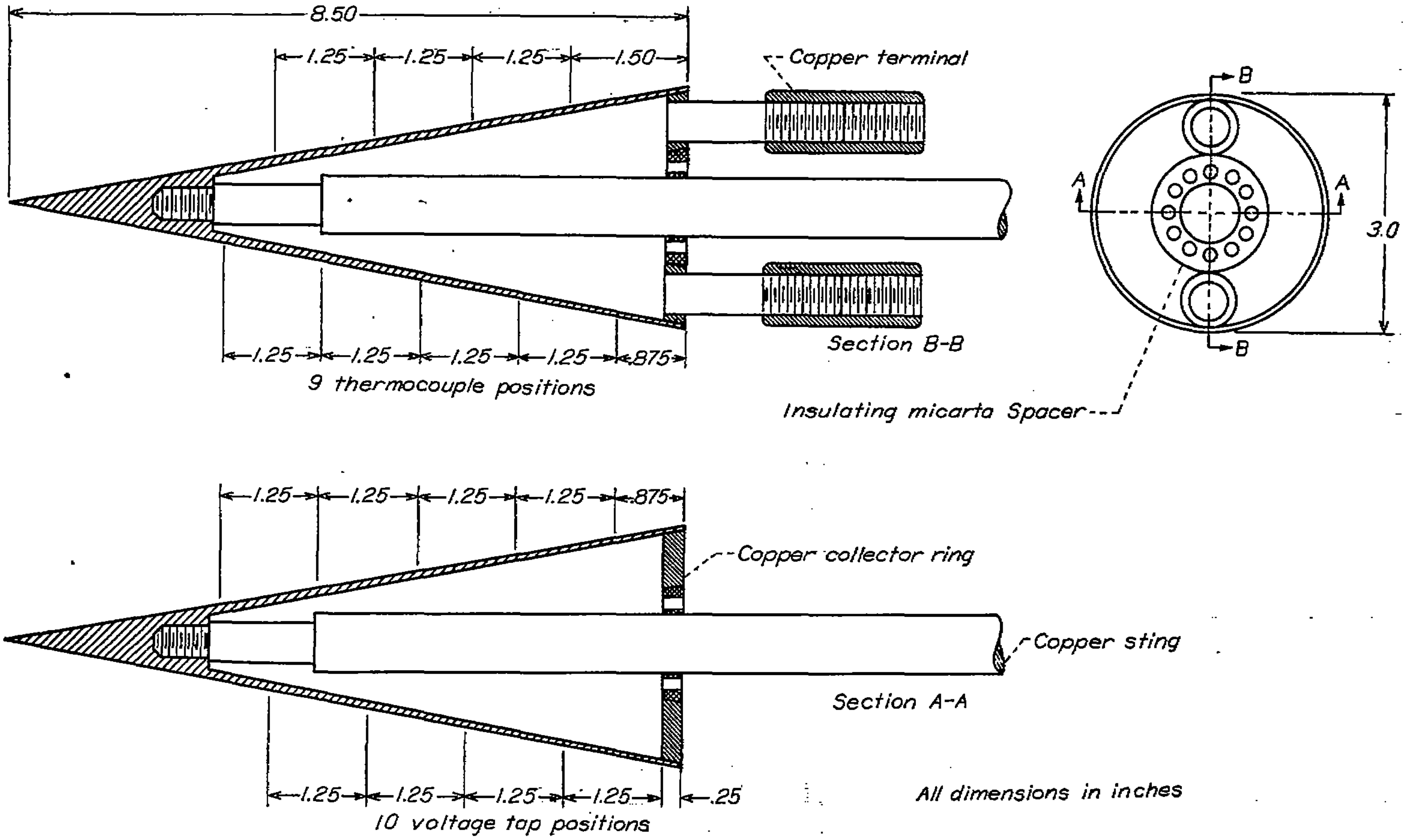


FIGURE 1.—Details of the electrically heated 20° cone.

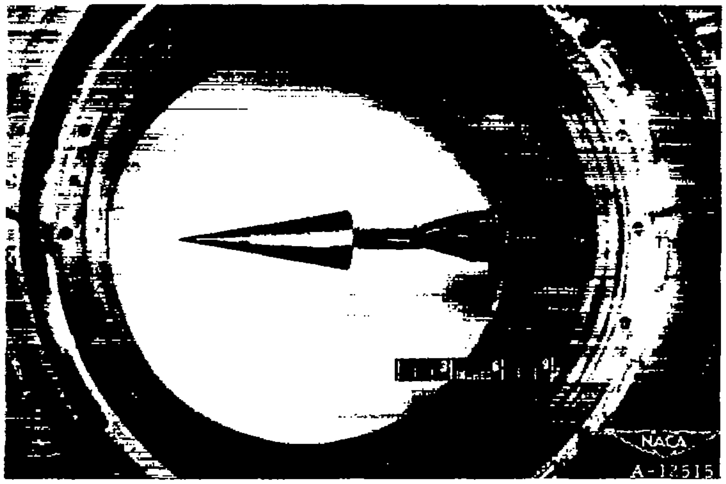


FIGURE 2.—Heated 20° cone installed in the test section of the 1- by 3-foot supersonic wind tunnel No. 1.

that of the recovery factor is ±1.5 percent. Because the over-all accuracy is largely controlled by the accuracy of the total-temperature measurements, the experimental scatter of data points obtained at a fixed value of the total temperature is much less than that indicated by the values of over-all accuracy. Although the small pressure gradient (± 1 percent of the average dynamic pressure) in the test section of the wind tunnel was neglected in the reduction of the test data, its effect has been included in the calculation of the maximum probable error.

An effort was made to determine the magnitude of the error due to heat radiation from the cone to the tunnel walls by experimental means. The radiant-heat loss was evaluated by the extrapolation to zero pressure of data obtained at several low values of total pressure with the wind tunnel in operation[3] and was found to be negligible.

COOLED-CONE TESTS

Air-cooled cone.—The construction details and the path of the cooling air through the cone are shown in figure 4. The outer shell was made of stainless steel and had a wall thickness of only 0.028 inch in order to minimize the heat conduction along the shell in the instrumented area. The inner cone also was made of stainless steel and had a wall thickness of 0.070 inch which was thin enough to make all conduction effects in the inner cone negligible. The annular gap between the cones was designed to give a uniform surface temperature in the instrumented region ($x^*=0.4$ to 0.8). Turbulent flow in this annular gap was necessary to obtain the most uniform distribution of coolant temperature across the gap; therefore, the internal cooling system was designed to operate at a Reynolds number sufficiently high to insure turbulent flow. In addition, the outer surface of the inner cone was roughened with $\frac{1}{32}$-inch-wide circumferential grooves spaced at $\frac{1}{4}$-inch intervals. During the investiga-

[3] An attempt was made to obtain the radiation calibration with the tunnel inoperative, but the cone surface temperatures were found to be very erratic because of free-convection currents. For this reason, the method was abandoned.

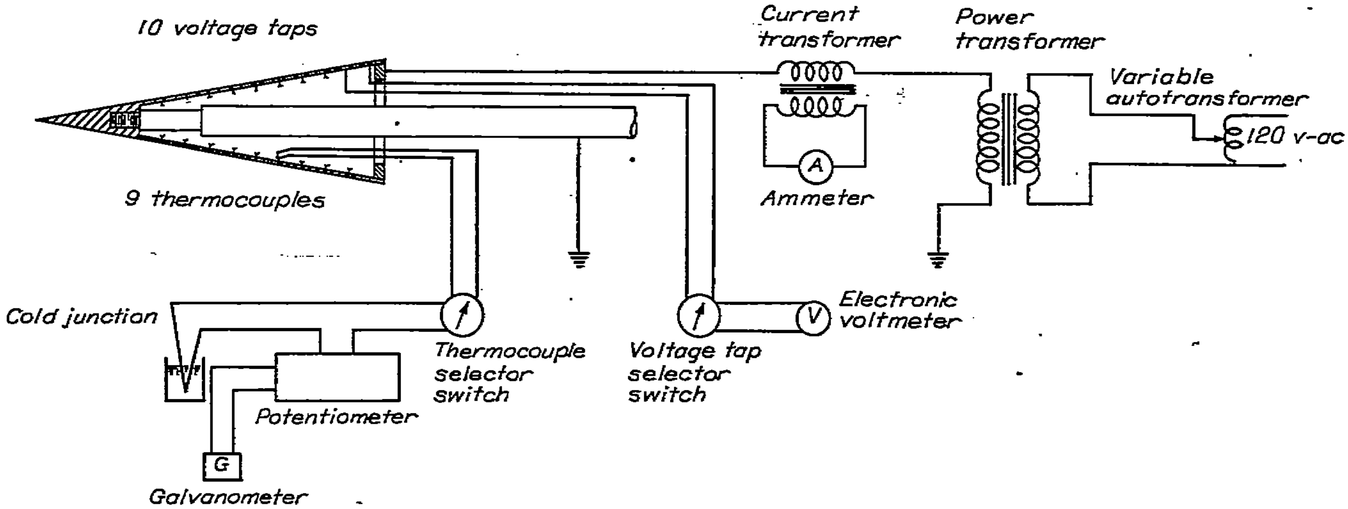


FIGURE 3.—Wiring diagram for the electrically-heated 20° cone.

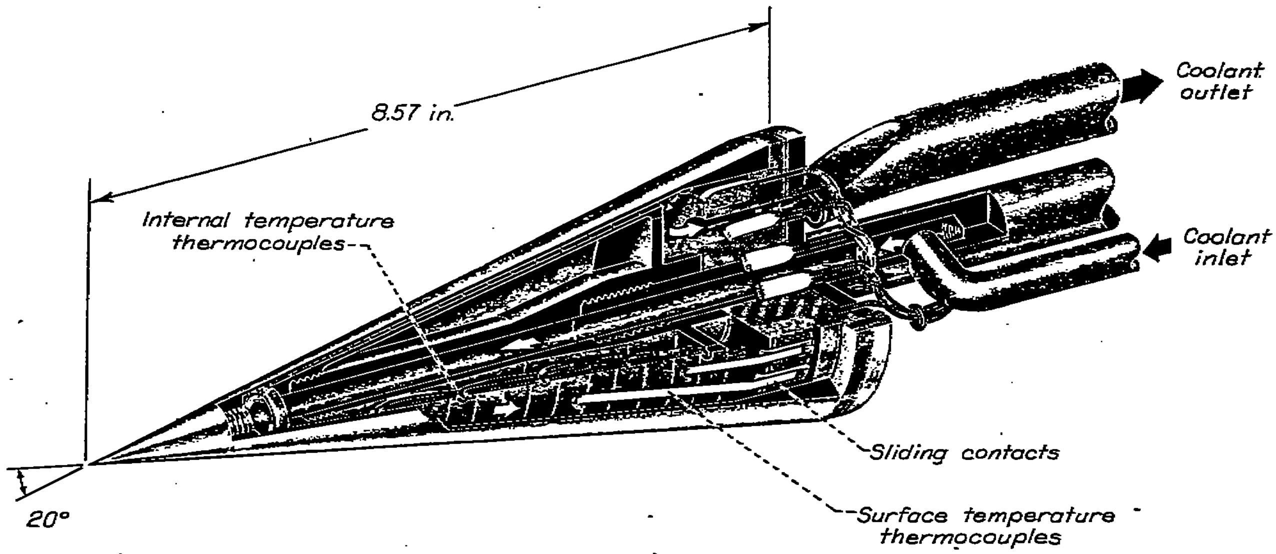


FIGURE 4.—Air-cooled cone.

tion, tests were made with coolant flow rates of 70 percent and 140 percent of the design value to determine the effect of changes in the internal-flow Reynolds number on the internal-temperature measurements. The results indicated that this effect was negligible.

Cooling system.—The primary requirement of the cooling system was that it provide an adjustable and stable outlet temperature at a constant coolant flow rate. This requirement was satisfied by the cooling system shown in figure 5. The clean, dry air which was used as the coolant in the cone was obtained from the make-up air system of the wind tunnel and could be cooled to a minimum temperature of −90° F. In the development of the cooling system it was found that the use of the recovery heat exchanger provided a marked increase in thermal stability. The temperature drift of the cooling system was extremely slow and amounted to less than ±0.1° F per minute.

Instrumentation.—Local rates of heat transfer were obtained by measuring the incremental temperature changes of a known weight-flow of coolant flowing along the annular gap between the inner and outer cones. The incremental coolant temperatures were measured by iron-constantan thermocouples spaced 1 inch apart along the annular gap. Three thermocouple junctions were located 120° apart midway between the walls of the gap at each of the five longitudinal stations. The surface temperature was measured with stainless-steel-constantan thermocouples consisting of the cone shell and thin (0.002 inch thick) constantan ribbons. The tips of the ribbons were soldered into holes in the shell. The thin strips were cemented to the inner surface of the shell. This installation minimized the interference of the constantan wires with the flow of air in the coolant passage. The flow rate of cooling air was measured with a rotameter located at the outlet of the cooling system.

Test procedure.—The tests were performed at three values of the wind-tunnel total pressure which gave length Reynolds numbers of 2.2, 3.6, and 5.0 million. Data were taken at the recovery temperature and at three nominal surface temperatures at each Reynolds number. The minimum surface temperature obtainable with the cooling system at each pressure, and temperatures approximately 20° F and 40° F above the minimum temperatures, were selected. The investigation was performed only at one Mach number (2.02) because the theory for heat transfer

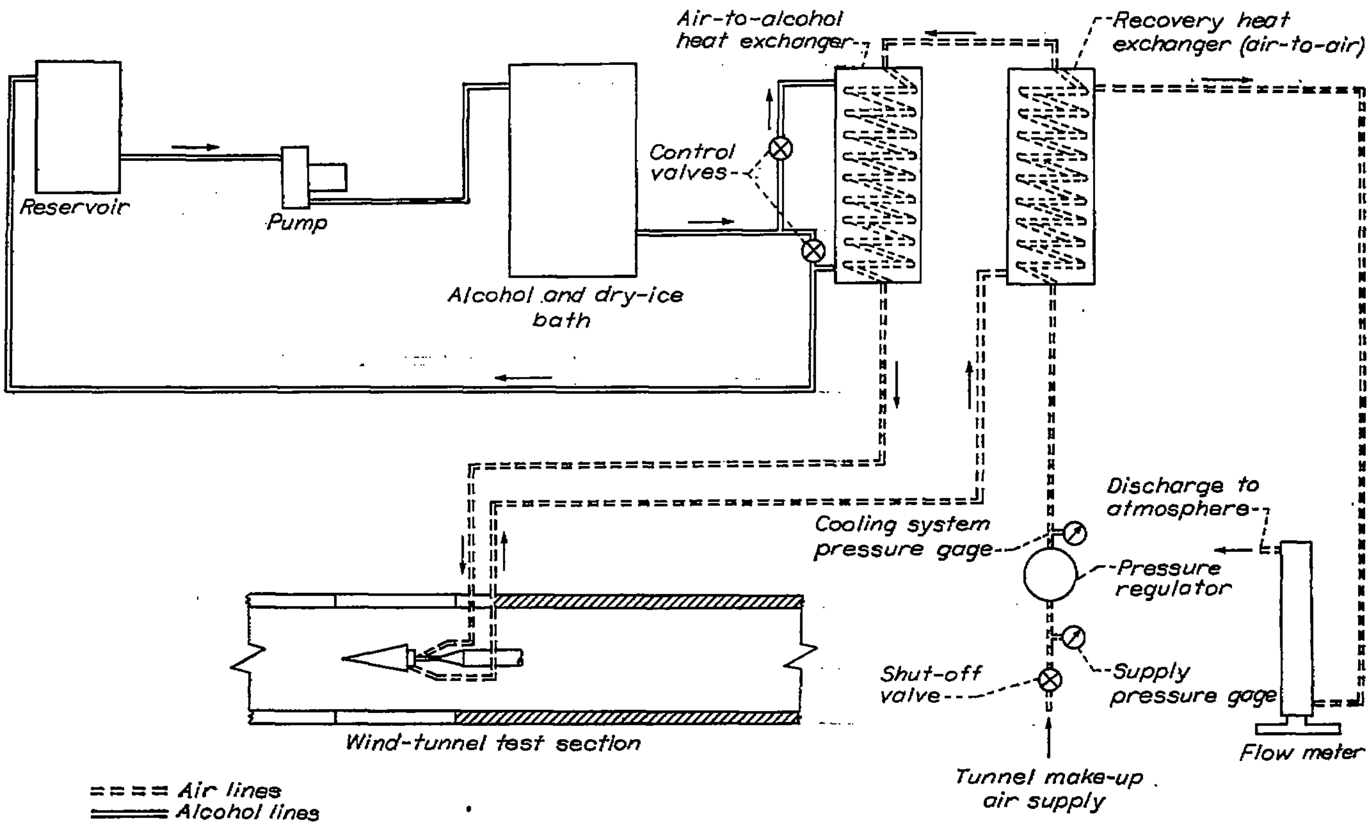


FIGURE 5.—Diagram of the air cooling system for the cooled 20° cone.

with uniform surface temperature and previous tests with the heated cone both indicated that the effect of Mach number on heat-transfer parameter is small within the range available in the wind tunnel (1.2 to 2.4). Schlieren observations, liquid-film tests, and the absence of discontinuities in the temperature-distribution measurements that would denote transition indicated that the boundary layer remained laminar for all test conditions.

In order to measure the effects of cooling on boundary-layer transition, it was necessary to have the transition point near the center of the instrumented area of the cone. However, it was known from the results of tests of the unheated cone that transition would occur downstream from the instrumented area at the condition of zero heat transfer at the maximum available Reynolds number. The required forward movement of transition was accomplished by the use of three small grooves around the cone at approximately the 8-, 10-, and 12-percent-length stations. All the grooves were 0.010 inch wide and 0.015 inch deep, and extreme care was taken to make them of uniform depth and with uniformly sharp edges in an attempt to obtain the same longitudinal position of transition on all rays of the cone. Data were obtained at several surface temperatures below the recovery temperature in successive decrements of about 10° F. The minimum surface temperatures investigated were the temperatures at which transition moved out of the instrumented area of the cone. The boundary-layer transition tests were conducted at a nominal length Reynolds number of 5.0 million at Mach numbers of 1.5 and 2.0.

Accuracy.—The accuracy of the final results is based on the accuracy of the individual measurements involved and on the probable uncertainty of some of the measurements due to peculiarities of the test apparatus. The estimated accuracy of the individual measurements in this test is as follows:

Total temperature, T_0	±1.5° F
Surface temperature, T_s	
At maximum T_s	±1.5° F
At minimum T_s	±4.5° F
Coolant-temperature increment, ΔT_s	
At maximum T_s	±0.07° F
At minimum T_s	±0.05° F
Internal air-flow rate, W (at design flow rate)	±1.4%
Cone dimensions	±0.002 in.
Cone-segment surface areas, A	1.4%
Total pressure, H	±0.01 psia
Static pressure, p	±0.01 psia

Although the surface-temperature thermocouples were calibrated and were accurate to within ±0.5° F, the experimental variations from curves faired through the sets of surface-temperature data were found to exceed this value. These variations are believed to have resulted from a nonuniform coolant distribution in the annular gap. The accuracy of the surface-temperature measurements at each set of test conditions was taken as the local difference between the faired curves and the data point farthest from the curves. The local values of surface temperature used in the reduction of the data were obtained from the faired curves.

Because the range of temperature differences between the cone and wind-tunnel-wall temperatures is similar to that in the heated-cone tests, the effect of radiant heat transfer on the experimental accuracy again has been neglected. The effects of longitudinal and circumferential heat conduction in the thin stainless-steel inner cone aft of the 50-percent-length station have been neglected. An approximate calculation of the actual conduction along the inner cone and through the inner-cone shell revealed that these effects were only about three-tenths of 1 percent of the total heat transfer. As in the case of the heated cone, the effect of the small static pressure gradient in the wind-tunnel test section has been neglected.

As in the heated-cone tests, the over-all accuracy of the final values of the heat-transfer parameter was calculated from the accuracy or uncertainty of each of the individual measurements for all test conditions by the method of reference 9. Because of the variation in the uncertainty of the surface temperature and internal-temperature increments, the final accuracy of the heat-transfer parameter varies from ± 12 percent at the most forward section of the cone to ± 8 percent at the rear of the cone.

PRESSURE-GRADIENT BODY TESTS

Body and instrumentation.—The body was axially symmetric and its radius at any longitudinal station is given by the relation

$$\frac{r}{L}=\frac{1}{3}\left[\left(\frac{x}{L}\right)-\left(\frac{x}{L}\right)^2\right] \tag{2}$$

Since the forward portion of the body was adequate for the purpose of the present investigation, the length L in equation (2) was assigned a value of 18 inches, but only the first 8½ inches of the total length were employed. The vertex angle of the test body was 37° and its fineness ratio was 2.83. Like the cones, the exterior shell of the body was machined from stainless steel. The heating circuit, instrumentation and general design of the parabolic-arc body were similar to those of the heated cone; details of the body are shown in figure 6. The shell thickness was designed to provide a uniform surface temperature at a Mach number of 1.5 by assuming that the theory for cones was applicable. In addition to the heated body, another body identical in contour was employed to determine the pressure distribution and, consequently, the Mach number distribution just outside the boundary layer along the body.

Procedure and accuracy.—The test procedure and accuracy with the parabolic-arc body were the same as those with the electrically-heated 20° cone. Data were obtained at nominal length Reynolds numbers of 2.5, 3.75, and 5.0 million at Mach numbers of 1.49 and 2.18. Surface temperatures of 120° F, 160° F, and 200° F were arbitrarily chosen as values at which to obtain data. Pressure-distribution measurements also were made at the same Mach numbers and total pressures as for the heat-transfer measurements. Because the local static pressures on the parabolic-arc body were measured, the maximum probable error of the recovery factor is ± 1 percent rather than ± 1.5 percent as in the case of the conical bodies.

RESULTS AND DISCUSSION

EFFECT OF HEAT TRANSFER ON BOUNDARY-LAYER TRANSITION

The effect on transition of adding heat to a laminar boundary layer at a Mach number of 1.99 is shown by the longitudinal surface-temperature distributions in figure 7. The effect of removing heat from a laminar boundary layer at Mach numbers of 1.50 and 2.02 is shown in figure 8. The start of transition is indicated by an abrupt decrease in surface temperature for the case of heat addition and by a rise in surface temperature with no heat transfer or when heat is removed from the boundary layer. These changes in surface temperature result from the difference in heat-

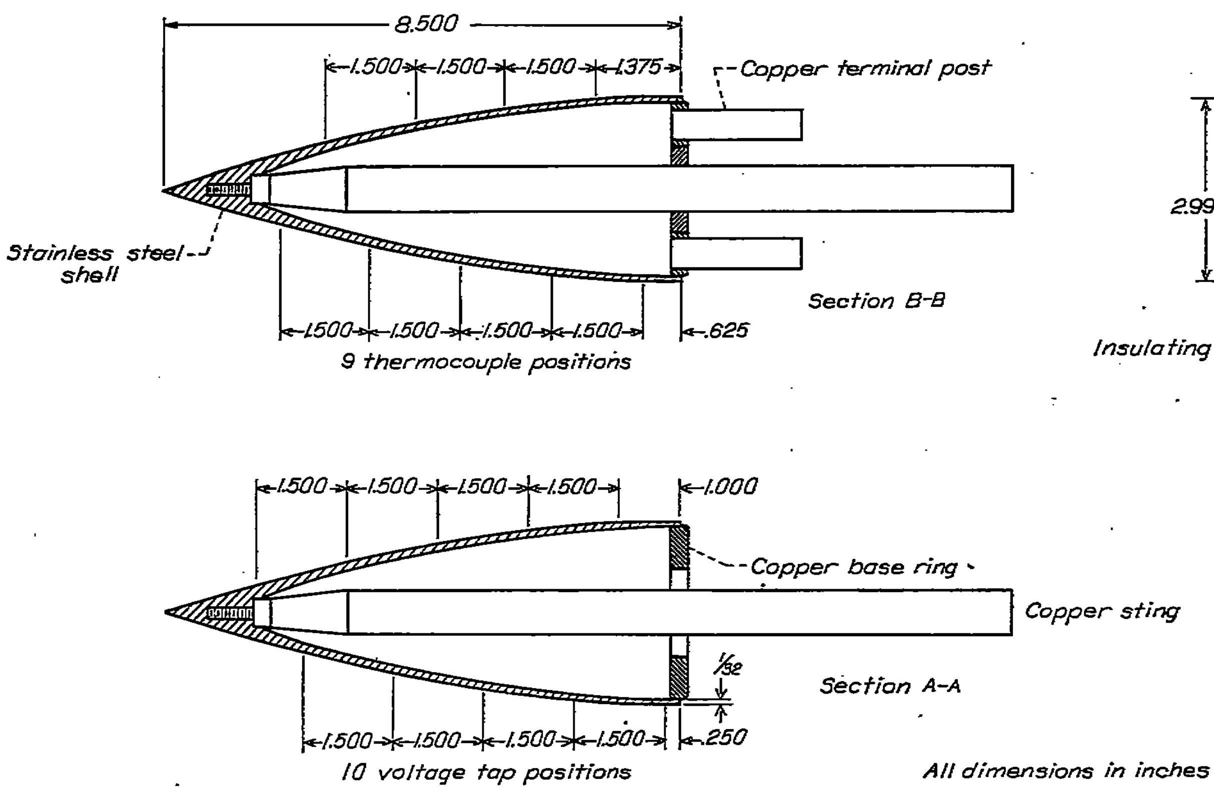


FIGURE 6.—Details of the electrically-heated parabolic body.

transfer coefficients between the laminar and turbulent boundary-layer regions. Only the data from one side of the cooled cone are presented in figure 8 because transition occurred in a similar manner on the opposite side, but at a position approximately 5 percent of the cone length aft of that shown at all test conditions. It should be noted that the lower curve in figure 7 and the upper curves in figure 8 were obtained at the condition of zero heat transfer ($T_s=T_r$).

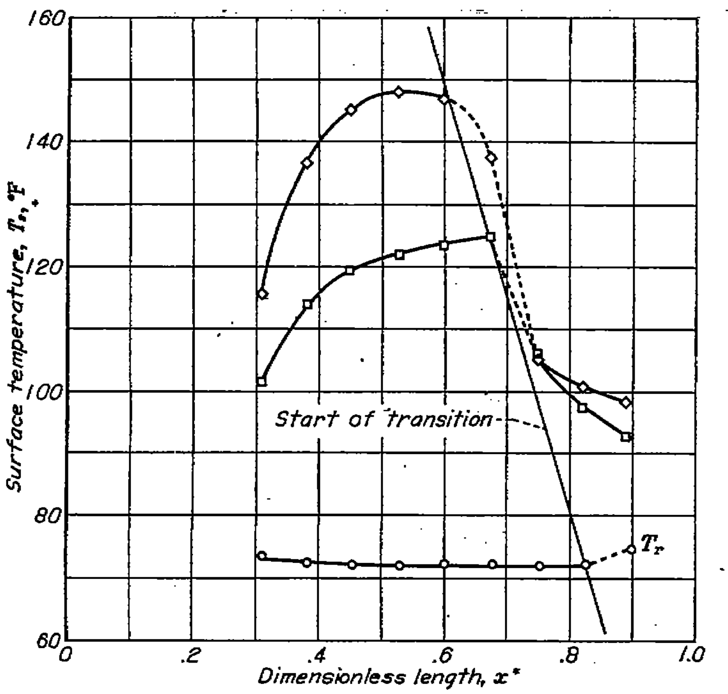
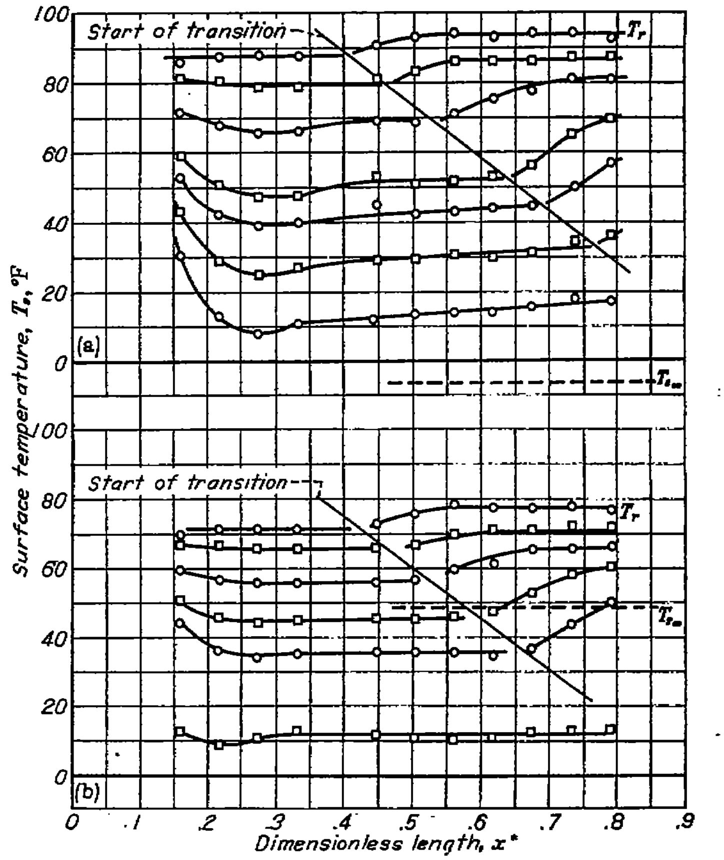


FIGURE 7.—The effect on transition of adding heat to the laminar boundary layer on a 20° cone; $M=1.99$, $Re_l=5.0\times10^6$.

In the case of the cooled cone (fig. 8), the fact that the rise in surface temperature is indicative of transition has been verified in four ways:

1. Liquid-film tests at the same length Reynolds number, at recovery temperature, have indicated transition in the region where the rise in surface temperature occurs.
2. The surface-temperature distributions ahead of the rise in surface temperature are almost identical in shape with those obtained in tests with the same body for which the measured heat transfer indicated laminar flow.
3. The region in which the rise in surface temperature occurs moved forward with an increase in the number of grooves.
4. The recovery factor increased by 0.04 in the region where the rise in surface temperature occurs,[4] which is in agreement with the change between laminar and turbulent flow obtained both in theory and in other experiments.

The variation of the extent of the laminar boundary layer along the heated and cooled cones indicates that heat addition promotes early transition while removal delays transition. The theory of reference 1 and these results are in qualitative agreement. For the present test conditions with the cooled cone, the theory indicates that the boundary layer should be infinitely stable at surface temperatures (T_{s_∞}) of −6.6° F and 49.0° F at Mach numbers of 1.5 and 2.0, respectively. The data shown in figure 8(b) show that transition occurred at surface temperatures below the theoretical infinite-stability limit at a Mach number of 2.02, and the data shown in figure 8(a) indicate that a similar result is possible at a Mach number of 1.50.



(a) $M=1.50$, $Re_l=5.0\times10^6$.
(b) $M=2.02$, $Re_l=5.0\times10^6$.

FIGURE 8.—Effect on transition of removing heat from a laminar boundary layer on a 20° cone with a small initial roughness.

The limited range of the present experiments makes it impossible to draw any definite conclusions regarding the quantitative effect of heat transfer or Mach number on transition. The data indicate, however, that the movement of transition with surface temperature for the cooled cone is at least similar at both Mach numbers. The constant slopes of the lines (in figs. 7 and 8) through the points at the beginning of transition for both cones indicate that, for the present experiments, the increase in the length of laminar run is directly proportional to the difference between the surface and recovery temperatures, and therefore is a linear function of the rate of heat transfer. Although the present experiments have proven qualitatively that heat transfer affects transition, additional experiments at higher Reynolds numbers (without roughness) and in a low-turbulence air stream are required to obtain quantitative data on the effect of heat transfer on boundary-layer transition.[5]

[4] The recovery factor on this model, without grooves, was 0.85, which is in agreement with the results for a laminar boundary layer obtained with the other models. The absolute values of the recovery factors ahead of and behind the rise in surface temperature were found to decrease continuously with an increase in number of grooves, but their difference was always 0.04±0.005. With the three grooves that were used, the laminar boundary-layer recovery factor was 0.81 and the turbulent boundary-layer value was 0.85. The reason for this change in recovery factor with number of grooves is unknown.

[5] A recent quantitative study of the effect of heat addition on transition has been made by Higgins and Pappas and published as NACA TN 2351.

RECOVERY FACTOR[4]

Typical values of the recovery factor obtained at several Reynolds numbers at nominal Mach numbers of 1.5 and 2.0 with the heated-cone model at the condition of zero heat transfer are shown in figure 9. Data obtained with the cone at a Mach number of 2.0 with various means of causing transition are also shown. Data obtained with the parabolic-arc body are shown in figure 10. The theoretical values indicated in both figures are based on a Prandtl number of 0.719 which is the value for dry air at 70° F. The theoretical recovery factors are therefore 0.848, $Pr^{1/2}$, for laminar boundary layers and 0.896, $Pr^{1/3}$, for turbulent boundary layers.

The experimental values of the recovery factor for turbulent boundary layers on the heated cone were obtained by resorting to artificial means of causing transition. Three methods were used—a ring of 0.005-inch diameter wire, a coat of lampblack-lacquer mixture, and a band of salt crystals. The wire and salt crystals were located at about 1 inch from the cone tip, and the lampblack covered the cone forward of the 1-inch position.

The data shown in figure 9 for laminar boundary layers are in fairly good agreement with theory. The difference between the values for the two Mach numbers in the region $x^*=0.3$ to 0.6 is believed to be partly due to the fact that the average Mach number was used in the reduction of the data. The most rearward data points obtained with a laminar boundary layer at the maximum Reynolds numbers (fig. 9) indicate the beginning of natural transition. The data obtained with turbulent boundary layers are somewhat lower than the theoretical value, but the aggreement is considered to be good enough to justify use of the theoretical value for design purposes. The differences between the results obtained with the various roughnesses decrease to small values toward the rear of the cone.

[4] A detailed discussion of the physical concept of the temperature recovery factor for laminar boundary layers is given by Wimbrow in NACA TN 1975.

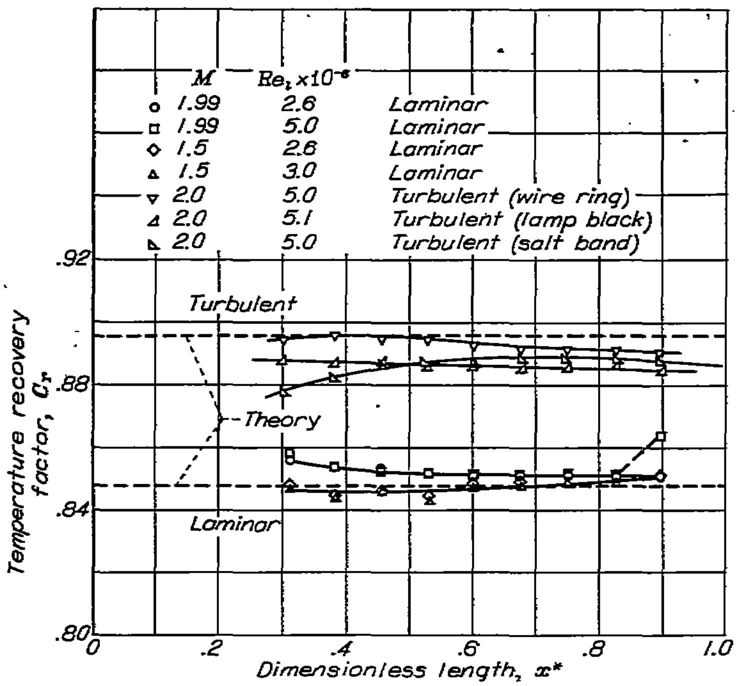
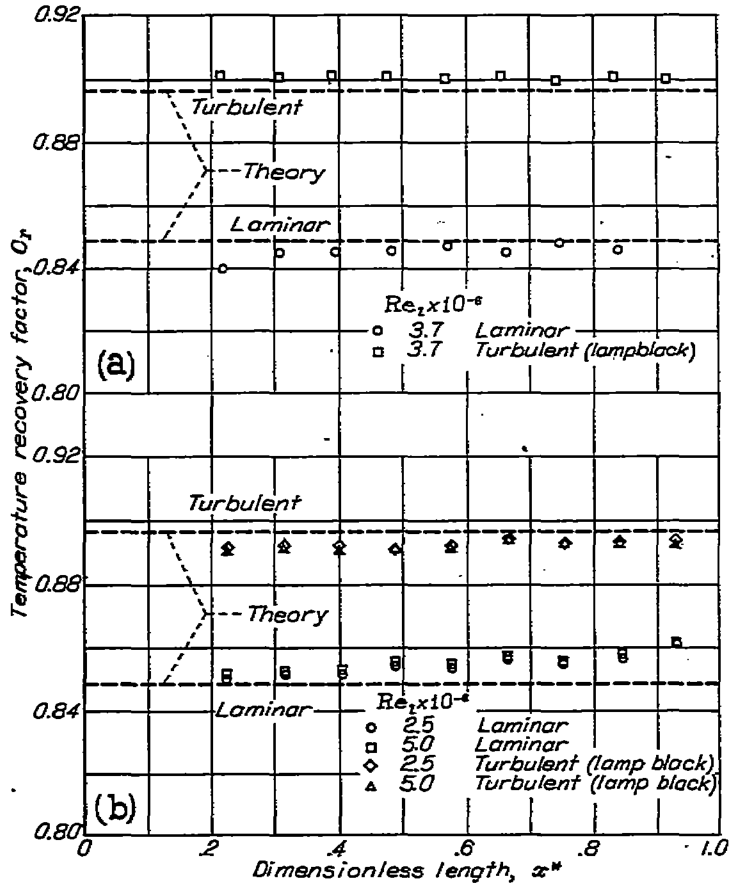


FIGURE 9.—Typical values of the temperature recovery factor for the 20° cone.



(a) $M=1.50$.
(b) $M=2.0$.

FIGURE 10.—Typical values of the temperature-recovery factor for the parabolic-arc body.

Since the external geometry and surface finish of the heated and cooled cones were identical, there were no essential differences in their recovery factors. For this reason, the recovery factors for the cooled cone are not presented.

The values of recovery factor for the parabolic-arc body with both laminar and turbulent boundary layers were based on the local Mach number just outside the boundary layer and these recovery factors are substantially constant along the body (fig. 10). The values of recovery factor for turbulent boundary layers, obtained by the use of lampblack, are somewhat greater than the values obtained with the cone (fig. 9). The data obtained with a laminar boundary layer on the parabolic-arc body at a Mach number of 2.0 show a gradual increase in recovery factor along the length of the body. This trend in the data is believed to have been caused by a gradual deterioration in the surface finish of the model due to testing and handling. Data obtained at a Mach number of 2.18 with this body when it was new and had a mirror-like finish indicated a constant recovery factor of 0.850. It is possible that surface roughness may cause some increase in the laminar boundary-layer recovery factor.

In general, the agreement between theory and experiment indicates that the recovery temperature on cones and in the negative-pressure-gradient region on bodies of revolution can be computed with satisfactory accuracy by use of the theoretical recovery factors and the local Mach number just outside the boundary layer.

HEAT TRANSFER

Results.—The theoretical values of the heat-transfer parameter for cones with laminar boundary layers and uniform surface temperatures are given by the following equation from reference 4:

$$\frac{Nu}{Re^{1/2}}=\frac{C_d\cdot Re^{1/2}}{2}Pr^{1/3}\sqrt{3} \tag{3}$$

in which the physical properties of the fluid are based on the temperature just outside the boundary layer. Crocco's theoretical calculations give values of the factor $C_d\cdot Re^{1/2}$ of 0.66 and 0.63 for small temperature differences ($T_s-T_r\cong$ 100° F) at Mach numbers of 1.5 and 2.0, respectively. If the Prandtl number is assumed to be 0.72, then the theoretical values of heat-transfer parameter at the two Mach numbers are 0.51 and 0.49.

The surface-temperature and heat-transfer-parameter data for the heated cone with an approximately uniform surface temperature at a Mach number of 1.50 are shown in figure 11 together with the theoretical value of the heat-transfer parameter for a uniform surface temperature (0.51). Similar data obtained at a Mach number of 1.99 for both the approximately uniform and the nonuniform surface temperatures are shown in figure 12, and the theoretical value of heat-transfer parameter for a uniform surface temperature (0.49) is shown for comparison. The data for the least uniform surface temperature shown in figure 12 is replotted in figure 13 and compared with a curve obtained by use of the theory of reference 5. The surface temperature distribution, converted according to the method of reference 8, is also shown in figure 13 and will be discussed later. The values of heat-transfer parameter for the nonuniform surface temperatures have been corrected for the effect of heat conduction in the shell due to the local surface temperature gradients. This correction for the approximately uniform surface temperatures was found to be negligible.

The heat-transfer data obtained with the cooled cone are

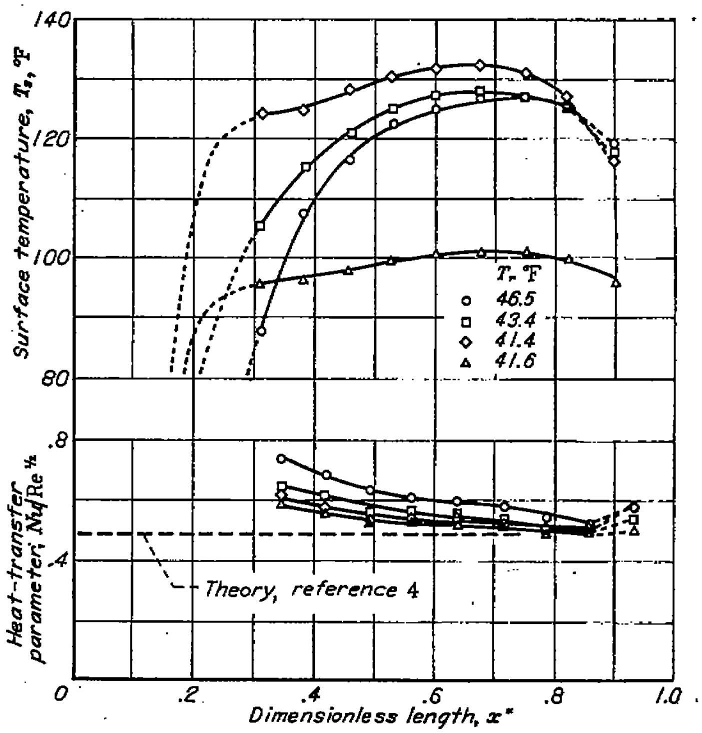


FIGURE 12.—Heat-transfer characteristics of the heated 20° cone with various surface-temperature distributions; M=1.99, Re_l=2.6×10⁶.

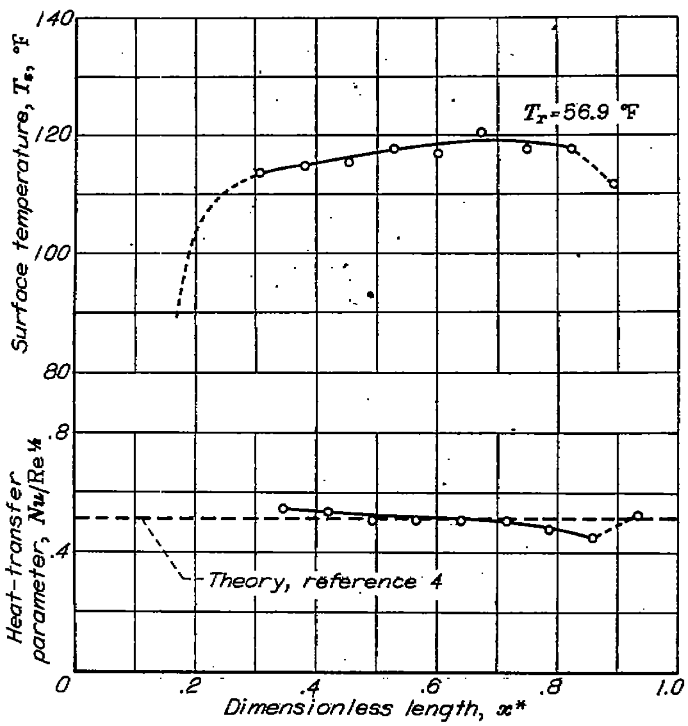


FIGURE 11.—Heat-transfer characteristics of the heated 20° cone with the most uniform surface temperature; M=1.50, Re_l=2.6×10⁶.

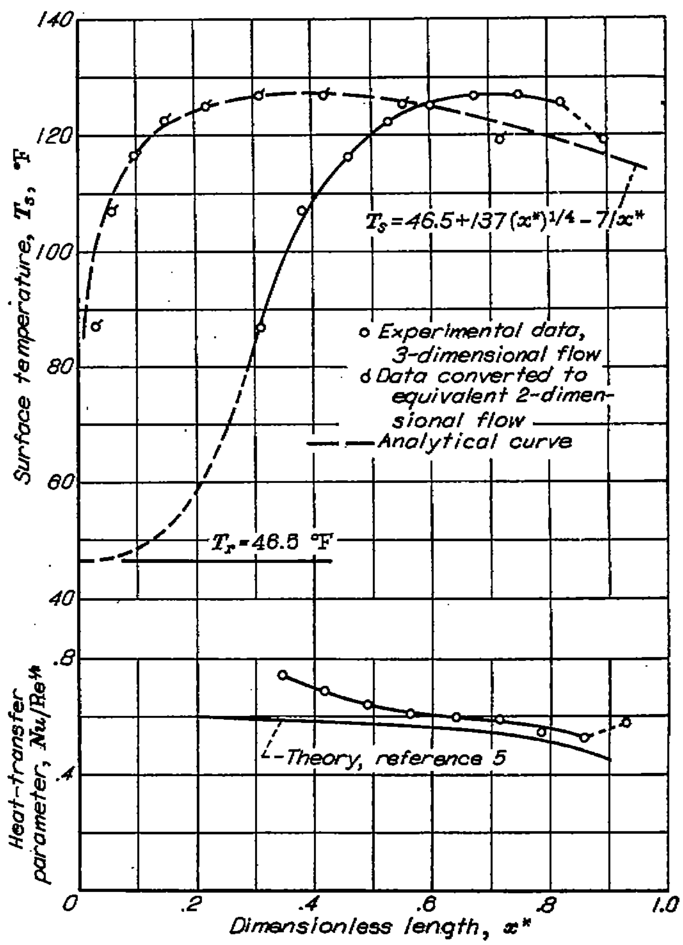


FIGURE 13.—Comparison of the theory for nonuniform surface temperatures with data from tests of the heated 20° cone; M=1.99, Re_l=2.6×10⁶.

presented in figure 14. All of the longitudinal surface-temperature distributions obtained during the tests were similar in shape, and therefore only three typical distributions, those for a Reynolds number of 2.2 million, are shown. Local values of heat-transfer parameter for the three surface temperatures at each of the three length Reynolds numbers are also presented. The surface-temperature gradient that existed over part of the segment nearest the tip resulted in a conduction loss of 3½ to 4 percent. The heat-transfer data has been corrected for this effect. Downstream of the first element there was no longitudinal conduction correction because the surface temperature in this region was nearly constant. The scatter in the experimental data (fig. 14) is about equal to the estimated maximum probable error which resulted from the inaccuracy of the measurements of the incremental changes in coolant temperature along the heat-transfer passage and from the scatter of the surface-temperature data. The theoretical value of the heat-transfer parameter, 0.49, is also shown in figure 14. The data nearest the nose deviate quite markedly from theory, a fact which is attributed to the effects of surface-temperature gradients on the laminar boundary layer. The values of heat-transfer parameter obtained with the heated cone with the approximately uniform surface temperature at a nominal surface temperature of 100° F (fig.12) have been plotted in figure 14 for comparison.

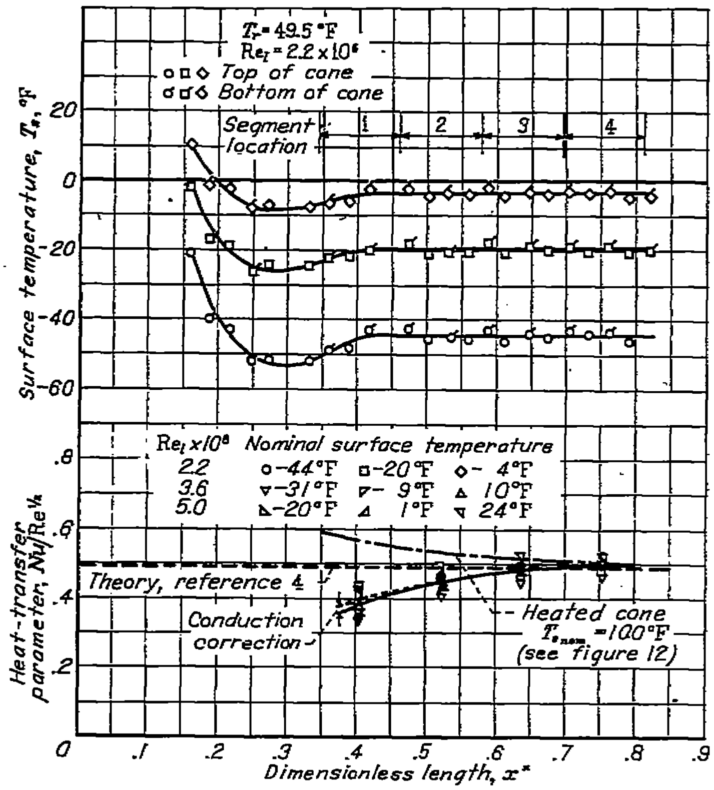


FIGURE 14.—Heat-transfer characteristics of the cooled 20° cone; M=2.02.

The measured pressure distributions required for the reduction of the heat-transfer data on the parabolic-arc body were reduced to the more convenient form of Mach number distributions and are shown in figure 15. The values of local Mach number were used to determine the local temperature at the outer edge of the boundary layer.

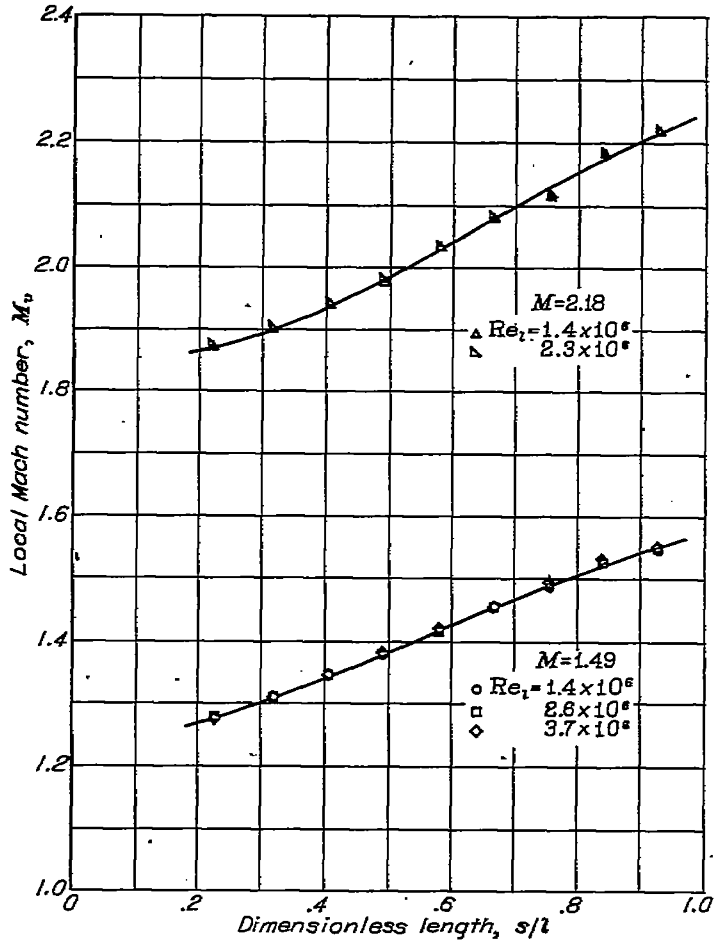
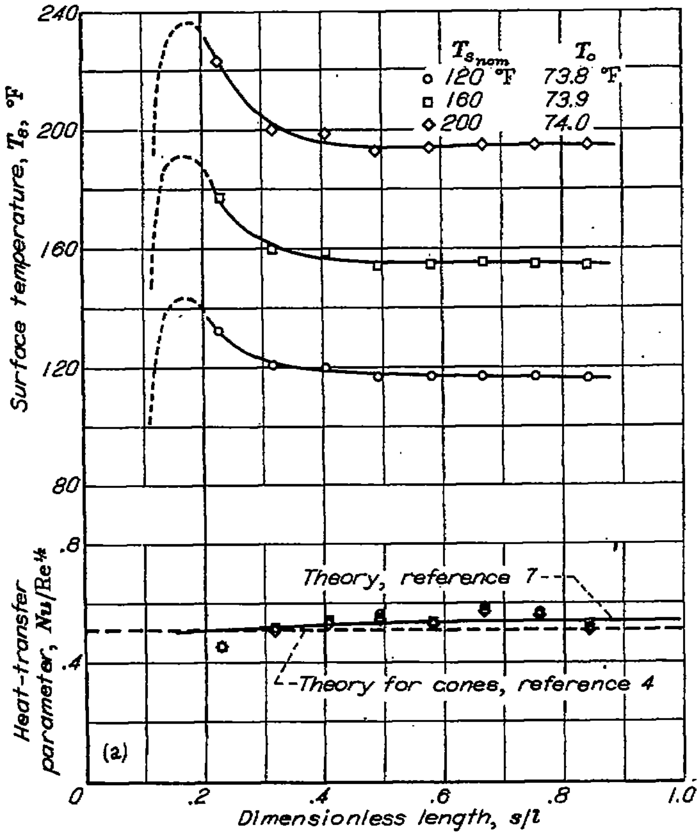


FIGURE 15.—Variation of local Mach number with length as determined by pressure measurements on the parabolic-arc body.

The heat-transfer data for the parabolic-arc body at a free-stream Mach number of 1.49 are shown in figure 16 and data obtained at a Mach number of 2.18 are shown in figure 17. Although heat-transfer data were obtained at three Reynolds numbers, 2.5, 3.75, and 5.0 million, at each Mach number, only representative data are presented in figures 16 and 17. The theoretical values of the heat-transfer parameter for cones are shown in figures 16 and 17 for comparison with the experimental data. In addition, local values of the heat-transfer parameter for a Mach number of 1.5 were computed for the test body by the method of reference 7. These values are shown in figure 16.

Inspection of the surface-temperature curves in figures 16(a), 16(b), and 17 reveals that the temperature at the first measuring station was considerably higher than the temperature at subsequent stations. After the tests were completed it was found that this local hot region was partially caused by poor electrical contact between the copper sting and the stainless-steel shell. This connection was improved and the body was tested again at a Mach number of 1.49. As shown in figure 16 (c), a much more uniform surface temperature was obtained.

Heat transfer with uniform surface temperature.—All three of the bodies tested had severe surface temperature gradients near the noses because the noses were solid and essentially unheated or uncooled. In the case of the heated bodies, the data obtained with approximately uniform



(a) Original surface-temperature distribution $Re_l=2.6\times10^6$.

FIGURE 16.—Heat-transfer characteristics of the parabolic-arc body; $M=1.49$.

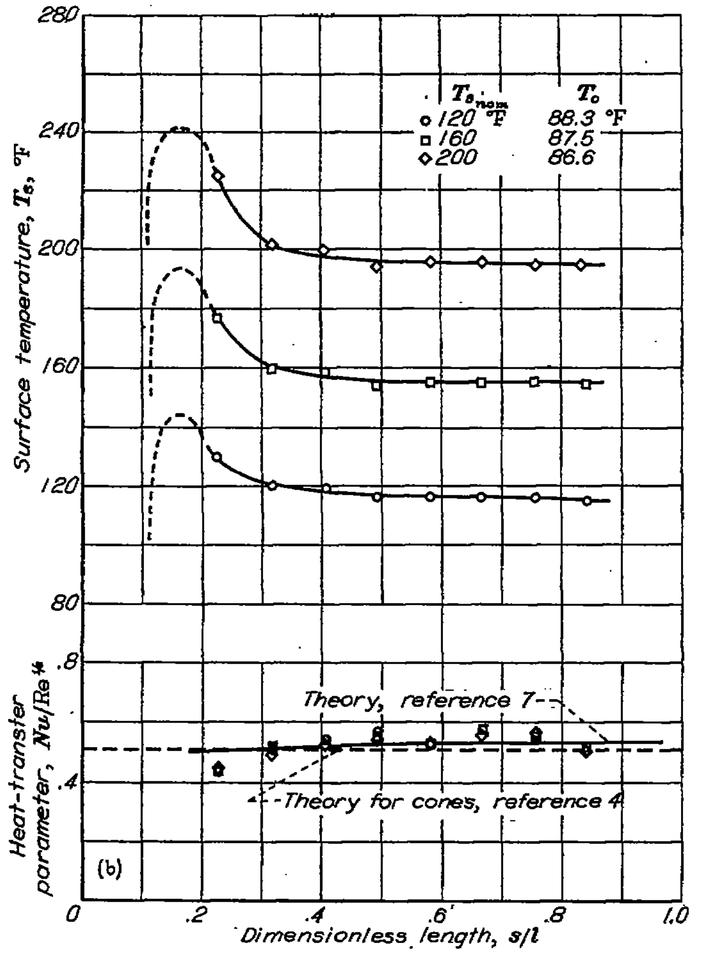


FIGURE 16.—Continued. (b) Original surface-temperature distribution $Re_l=3.7\times10^6$.

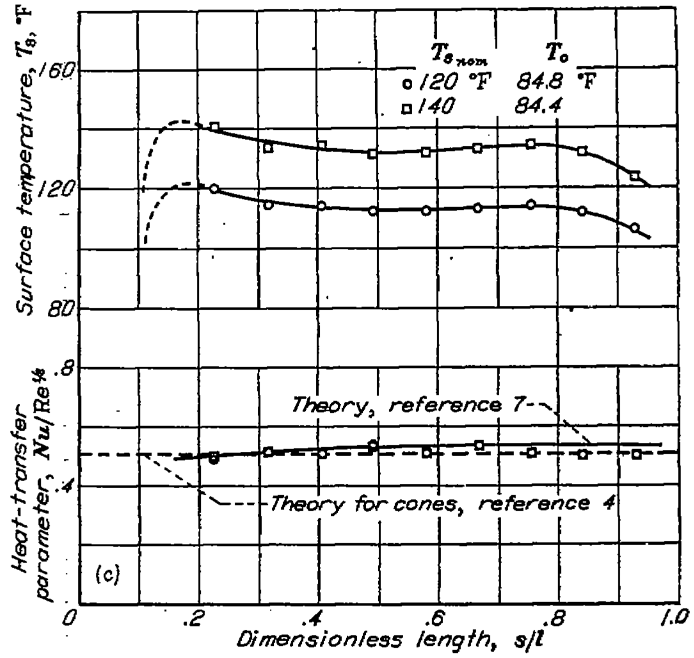


FIGURE 16.—Concluded. (c) Modified surface-temperature distribution $Re_l=2.6\times10^6$.

surface temperatures (figs. 11, 12, and 16 (c)) indicate that when the large initial surface-temperature gradient is confined to the nose region on a body the effect on the local values of the heat-transfer parameter is small downstream of the initial gradient. In the case of the parabolic-arc body which had the most confined initial gradient region ($x^*=0$ to 0.16), the data (fig. 16 (c)) indicate that the effect at the most forward data point $x^*=0.23$ was almost negligible.

The fact that the effect of a surface-temperature gradient is dependent on its lengthwise location on the body can also be shown by the theory of reference 5. In this theory the surface-temperature distribution is expressed in a polynomial form such as the following:

$$T_s^*=a_0+a_1x^*+a_2x^{*2}\ \ldots\ a_nx^{*n} \tag{4}$$

The heat-transfer parameter for cones is then given by the following equation:

$$\frac{Nu}{Re^{1/2}}=-\frac{1}{2}\sqrt{3}\left[\sqrt{\frac{(T_s+460)}{(T_r+460)}}\ \frac{(T_r+460)+216}{(T_s+460)+216}\right]\frac{\sum_{n=0}^{n=\infty} a_nx^{*n}\,Y_n'(0)}{T_s^*-T_r^*} \tag{5}$$

where a and n are coefficients and exponents, respectively, as shown in equation (4) and the values of $Y_n'(0)$, taken from reference 5, are given in the following table:

n	$Y_n'(0)$
0	−0.5915
1	−.9775
2	−1.1949
3	−1.3680
4	−1.4886
5	−1.5975
10	−2.0121

Although only the values of $Y_n'(0)$ for positive integer values of n are tabulated, it has been found that forms of equation (4) employing fractional exponents are particularly useful in reducing the required number of terms in polynomials to give certain surface-temperature distributions. Since polynomials with fractional exponents satisfy the basic differential equations of reference 5, the use of fractional-exponent terms in equation 5 is acceptable. The appropriate values of $Y_n'(0)$ can be obtained for any fractional value of n by interpolation between the values tabulated above.

To illustrate that the effect of a surface-temperature gradient depends upon its position along the body, assume a number of separate surface-temperature distributions all given by the equation

$$T_s^*=a+b(x^*)^n \tag{6}$$

where, for a given distribution, $n=1$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, or zero. In going from the larger to the smaller values of n, the region of the large initial temperature gradients for each curve will be confined to a smaller and smaller region near the origin. It is obvious that at $n=0$ the surface temperature will be uniform. Now, turning to the values of $Y_n'(0)$, it can be seen that the values decrease as n decreases. Therefore, the heat-transfer parameter curves (equation (5)) for each successively smaller value of n approach the constant value for a uniform surface temperature ($n=0$). This is the effect that was observed in the experiments. (See fig. 12.)

In the case of the cooled cone, the region of nonunifrom surface temperature extended to the 40-percent length station, and the surface-temperature gradient reversed in sign between the 20- and 40-percent length stations. (See fig. 14.) It might be expected from the theory of reference 5 that the lag in the boundary-layer temperature profile could result in a local decrement in the heat-transfer parameter when the surface-temperature gradient reversed. From the previous discussion, where it was shown that the effects of a surface-temperature gradient decrease downstream of the gradient, it would be expected that the agreement between the experimental and theoretical values of heat-transfer parameter would improve toward the rear of the body. These effects are shown by the data of figure 14. It is interesting to note that the local hot region ($x^*=0.2$) on the parabolic-arc body caused surface-temperature distributions similar to those of the cooled cone. (See figs. 14, 16(a), 16(b), and 17.) The effect of the reversal in surface-temperature gradients again caused the local values of heat-transfer parameter to be less than the "uniform-temperature" value.

The curve representing the heat-transfer data from tests of the heated cone with the most uniform surface temperature (see fig. 12) has been replotted in figure 14. As can be seen, in the regions where the effects of the initial surface-temperature gradient are small, the results from tests of the heated and cooled cones are in good agreement. It can be concluded that for the case of laminar boundary layers on bodies with uniform pressures and uniform surface temperatures, the available theory is satisfactory for engineering purposes.

Heat transfer with nonuniform surface temperatures.—The data obtained from tests of the heated cone with the least uniform surface-temperature distribution are replotted in

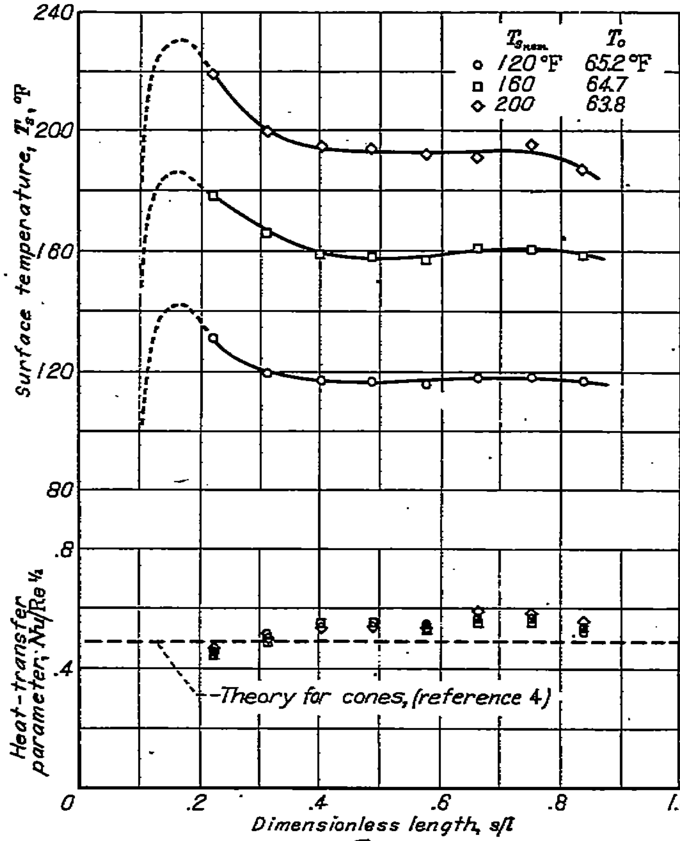


FIGURE 17.—Heat-transfer characteristics of the parabolic-arc body with the original surface-temperature distribution; $M=2.18$, $Re_l=2.6\times10^6$.

figure 13 for comparison with the theory of reference 5. (See equation (5) of the present report.) The first step in this comparison was the conversion of the experimental surface-temperature distribution from that of a cone to the equivalent distribution on a flat plate. Mangler, reference 8, has shown that the conversion is:

$$(x^*_{cone})^3=x^*_{plate} \tag{7}$$

The converted distribution is shown by the "flagged" symbols in figure 13. The second step in the comparison was to determine the values of the coefficients a_0, a_1, a_2, . . . a_n and values of the exponents of equation (4) so that the equation would fit the converted surface-temperature distribution. It has been assumed from the shape of the curve through the experimental surface-temperature data shown in figure 13 that the dashed extension of the curve in the region $x^*=0$ to 0.3 is a satisfactory approximation to the actual surface-temperature distribution. Although this assumed portion of the curve and the analytical curve may both be in error in the region $x^*=0$ to 0.1 on the converted scale, it has already been shown that initial temperature gradients are not critical in determining heat transfer a short distance downstream from the gradients. For this reason, and others that will be discussed later, an expression was derived that fits the data quite accurately only from $x^*=0.1$ to 0.7.

This equation in dimensionless form is

$$T_s^*-T_r^*=0.425\,(x^*)^{1/4}-0.22\,x^* \tag{8}$$

The local heat-transfer parameters obtained by using equations (5) and (8), and converted back to the x^* values for cones, are shown in the lower half of figure 13. Agreement between theory and experiment would not be expected ahead of the 47-percent length station on the experimental curve because of the increasingly poor fit of the experimental and analytical temperature curves ahead of the 10-percent length station on the converted curve ($0.10^{1/3}=0.47$, equation (7)). However, the theory of reference 5 and the experiments are in quite good agreement downstream of the 47-percent length station.

The abrupt rise in heat-transfer parameter at the rear of the heated cone as shown in figures 11, 12, and 13 ($x^* = 0.93$) is not due to surface-temperature gradient effects, but is due to the beginning of transition.

In the preceding discussion it was necessary to accept a poor fit between the data and the analytical surface-temperature distribution (equation (8)) in the region $x^*=0$ to 0.1. Considerable difficulty has been experienced in obtaining sufficiently accurate analytical expressions for the surface-temperature distributions. The obvious approach to obtaining polynomial expressions is the method of least squares or some other method of curve fitting using real, positive integers as exponents. In general, the analytical curves obtained with such methods oscillated about the experimental curve and near $x^*=1.0$ abruptly diverged due to the influence of the last term in the polynomial. It was found that the alternating positive and negative surface-temperature gradients due to the oscillation of the analytical curves resulted in even greater relative oscillations in the corresponding calculated heat-transfer-parameter curves. Also, because of the effect of the values of Y_n' (0) on the coefficients of the polynomial, the abrupt divergence of the heat-transfer parameter resulting from the divergence of the temperature occurred at values of x^* below 1.0. It appears that any satisfactory polynomial with integer exponents must have a large number of terms and must give a fair curve to well beyond $x^*=1.0$. The coefficients of such polynomials are tedious to determine. An alternative procedure which was used in the present investigation, consisted of using a fairly simple analytical expression to obtain a good fit between the curves over the major part of the cone length while accepting differences in the nose region. Although the results for the particular case shown in figure 13 indicate agreement between theory and experiment, downstream of the 47-percent length station, it is evident from the previous discussion that the restrictions imposed by the form of equation (4) limit the application of the theory of reference 5 to only simple variations of surface temperature with body length.

Heat transfer on a body with nonuniform pressure.—The boundary layer on a flat plate grows only in thickness with distance along the surface, while the boundary layer on a pointed body of revolution must spread circumferentially as it grows in thickness. The effect of this circumferential spreading on the boundary-layer thickness, and hence on the heat transfer, varies with the rate of change of circumference with respect to length and can be evaluated for any fair contour by relations given in reference 8. For example, it can be shown that on a cone, for which the pressure and the rate of change of circumference are constant with length, the boundary-layer thickness is less than that on a flat plate by the factor $1/\sqrt{3}$ and the rate of heat transfer is greater by the factor $\sqrt{3}$. For a parabolic-arc body, the factors defining the boundary-layer thickness and the rate of heat transfer relative to a two-dimensional surface with an equivalent pressure distribution vary from the conical values ($1/\sqrt{3}$ and $\sqrt{3}$, respectively) at the vertex to unity at the point where the rate of change of circumference becomes zero. Therefore, the rate of growth of the boundary layer and the rate of heat transfer on such a body are the same as on a cone at the apex, but the decreasing rate of change of circumference along the body tends to increase the rate of growth of the boundary layer relative to that on a cone and tends to decrease the rate of heat transfer. Although the effect of a pressure gradient on the characteristics of a boundary layer cannot be predicted exactly, it is known that a negative pressure gradient causes the boundary layer to thicken less rapidly with length than is the case if the pressure were constant, regardless of the body shape. Thus, on a body for which both the pressure and the rate of change of circumference decrease with length, the effect of the pressure gradient tends to counteract the effect of the variation of circumference on the boundary-layer characteristics.

The heat-transfer data obtained with the parabolic-arc body and the theoretical values of heat-transfer parameter for uniform-temperature cones (0.51 at a Mach number of 1.49 and 0.48 at a Mach number of 2.18) are shown in figures 16 and 17. It is apparent in figure 16 that the data, the theory for cones, and the method of reference 7, are in fairly good agreement. After the local hot region at the nose of the body had been eliminated, the data shown in figure 16 (c) were obtained. Good agreement between the experimental data, the theory for cones, and the method of reference 7, is again apparent. Because of the good agreement between the theory for cones and the experimental data shown in figure 16 (c), and the agreement shown in figures 16 (a), 16 (b), and 17, it can be expected that when the surface temperature is uniform, good agreement would be obtained at least throughout the test range of Mach number and Reynolds number. It should be noted that the small variation of the individual values of heat-transfer parameter from a fair curve is consistent throughout and is possibly due to small deviations in thickness of the cone shell. Comparison of the data of figure 16 (c) with the data of figures 16 (a), 16 (b), and 17 reveal that there was a slight reduction in the local values of the heat-transfer parameter over most of the body when the negative temperature gradient at the nose was reduced. However, the reduction at each station on the body where it occurred is within the experimental accuracy.

Since the experimental values of the heat-transfer parameter remained essentially constant along the parabolic-arc body, the effect of the negative pressure gradient is apparently opposite and, approximately equal to the effect of the decreasing rate of change of the circumference on the growth of the boundary layer. The present investigation, however, is inconclusive as to the exact individual effects of the pressure gradient and body shape on heat transfer. It has been shown that the local heat transfer for a parabolic-arc body can be calculated with satisfactory accuracy by applying the theory for cones. Because of the agreement between the results of the experiments, the method of reference 7, and the theory for cones, it appears reasonable to assume that this agreement would be obtained with other pointed body shapes that give rise to uniform negative-pressure gradients. On a body that is more slender than the one tested, the effects of the changing circumference and the pressure gradient would be less and would still tend to counteract each other. Conversely, on a more blunt body both effects should be larger. However, additional experiments with other body shapes covering a wider range of Mach numbers are necessary before the theory for cones can be considered to be applicable to all fair bodies of revolution with negative pressure gradients.

CONCLUSIONS

The results obtained from tests of three bodies of revolution at supersonic velocities have provided information on three important phases of the heat-transfer problem—boundary-layer transition, temperature-recovery factor, and heat-transfer parameter (for laminar boundary layers). The following conclusions are based on these results and comparisons with available theories:

1. The effect of adding heat to a laminar boundary layer is to cause premature transition and the effect of extracting heat is to delay transition although to lesser degree than could be expected from the implications of the theory of Lees (NACA Rep. 876).

2. The recovery temperature on cones and in the negative pressure-gradient region on bodies of revolution can be computed with satisfactory accuracy by use of the theoretical recovery factors for laminar and turbulent boundary layers and the local Mach number just outside the boundary layer.

3. The values of heat-transfer parameter for laminar boundary layers obtained from the theories for uniform surface temperatures (Crocco, and Hantsche and Wendt) are in good agreement with experiment for both heated and cooled bodies.

4. Values of heat-transfer parameter for laminar boundary layers obtained from the theory of Chapman and Rubesin for nonuniform surface temperatures are in good agreement with experiment; however, the agreement is markedly dependent upon obtaining an exact analytical expression for the experimental surface-temperature distribution.

5. The values of heat-transfer parameter on the forward half of a pointed parabolic-arc body with a laminar boundary layer when based on the local Mach number just outside the boundary layer are almost identical to those of cones.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., *December 17, 1948.*

REFERENCES

1. Lees, Lester: The Stability of the Laminar Boundary Layer in a Compressible Fluid. NACA Rep. 876, 1947. (Formerly NACA TN 1360.)
2. Van Driest, E. R.: Turbulent Boundary Layer in Compressible Fluids. Jour. Aero. Sci., vol. 18, no. 3, March 1951, pp. 140–161.
3. Stalder, Jackson R., Rubesin, Morris W., and Tendeland, Thorval: A Determination of the Laminar-, Transitional-, and Turbulent-Boundary-Layer Temperature-Recovery Factors on a Flat Plate in Supersonic Flow. NACA TN 2077, 1950.
4. Johnson, H. A., and Rubesin, M. W.: Aerodynamic Heating and Convective Heat Transfer—Summary of Literature Survey. Trans. ASME, vol. 71, no. 5, July 1949, pp. 447–456.
5. Chapman, Dean R., and Rubesin, Morris W.: Temperature and Velocity Profiles in the Compressible Laminar Boundary Layer with Arbitrary Distribution of Surface Temperature. Jour. of Aero. Sci., vol. 16, no. 9, Sept. 1949, pp. 457–565.
6. Kalikhman, L. E.: Heat Transmission in the Boundary Layer. NACA TM 1229, 1949.
7. Scherrer, Richard: The Effects of Aerodynamic Heating and Heat Transfer on the Surface Temperature of a Body of Revolution in Steady Supersonic Flight. NACA Rep. 917, 1948. (Formerly TN 1300, 1947.)
8. Mangler, W.: Compressible Boundary Layers on Bodies of Revolution. M. A. P. Volkenrode, VG83 (Rep. and Trans. 47T) Mar. 15, 1946.
9. Michels, Walter C.: Advanced Electrical Measurements. D. Van Nostrand Company, Inc. (New York) 2d ed., ch. 1, 1943.